

A Fuzzy Set Approach to Modifiers and Vagueness in Natural Language

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SUMMARY

Recent developments in semantic theory, such as the work of Labov (1973) and Lakoff (1973), have brought into question the assumption that meanings are precise. It has been proposed that the meanings of all terms are to a lesser or greater degree vague, such that, the boundary of the application of a term is never a point but a region where the term gradually moves from being applicable to nonapplicable.

Developments in fuzzy set theory have made it possible to offer a formal treatment of vagueness of natural language concepts. In this article, the proposition that natural language concepts are represented as fuzzy sets of meaning components and that language operators—adverbs, negative markers, and adjectives—can be considered as operators on fuzzy sets was assessed empirically. In a series of experiments, we explored the application of fuzzy set theory to the meaning of phrases such as *very small*, *sort of large*, and so on.

In Experiment 1, subjects judged the applicability of the set of phrases to a set of squares of varying size. The results indicated that the group interpretation of the phrases can be characterized within the framework of fuzzy set theory. Similar results were obtained in Experiment 2, where each subject's responses were analyzed individually. Although the responses of the subjects, in general, could be interpreted in terms of fuzzy logical operations, one subject responded in a more idiomatic style.

Experiments 3 and 4 were attempts to influence the logical-idiomatic distinction in interpretation by (a) varying the presentation mode of the phrases and by (b) giving subjects only a single phrase to judge.

Overall, the results were consistent with the hypothesis that natural language concepts and operators can be described more completely and more precisely using the framework of fuzzy set theory.

Picture a conversation between a mother and her 5-year-old son. The son asks the mother if he might have some jelly beans from the bowl on the table. The mother replies that he may take a few, and he does just that. It is a perfectly normal conversation until one stops and considers how many *a few* are. The mother obviously knows what is meant by the term *a few*. The information was understood by her son because he responded correctly to the

mother's statement. If the son had been asked directly how many constitute *a few*, he probably would have hesitated for a moment and then responded "three" (or "four," or "five," and so on). Pose the same question to the mother, and the odds are that she would have responded with not too different a number. The number of beans taken might have differed from what either person might have replied, but the actual number was probably an acceptable

definition from the mother's point of view. If it were not, the son certainly would have heard about it!

This scene demonstrates the transmission of some vague, quantitative information between two people. But actually, when one stops and considers, most of the quantitative information that one receives during the course of a day is of just this nature. A house may be *quite large*; a girl may be *sort of short*; a man may be *not very old*. Not only do people understand such statements, they also have the ability to operate upon and manipulate these vague concepts.

Recently, there has been considerable interest on the part of linguists and computer scientists in just such problems as the role of vagueness in language and the quantification of meaning. Much of this interest has been the result of the development of fuzzy set theory, a generalization of the traditional theory of sets. A major feature of fuzzy set theory is that a quantitatively specifiable system can contain linguistic variables in addition to numeric variables. These linguistic variables can be manipulated and operated upon in much the same way as numeric variables in nonfuzzy systems.

This new way of dealing with complex systems appears quite promising in terms of the specification of complex behavioral processes, such as the measurement of word meaning or the description of reasoning processes in everyday situations. In this article, we explore the possibilities of developing a treatment of (a) natural language concepts as fuzzy sets and (b) modifiers as operators on fuzzy sets.

Quantification of Meaning

One of the first attempts at the quantification of meaning was made by Mosier (1941). Mosier hypothesized that the meaning of a word may be considered as containing two components: (a) a constant component reflecting the overall meaning value along a continuum and (b) a variable component representing the variation in the meaning of the word due to context, speaker, and the like. He defined the meaning (M) of a word as:

$$M = x + i + c,$$

where x equals the constant component over people and context, i equals the variation in meaning due to the individual, and c equals the variation in meaning due to the context.

According to this theory of word meaning, any one of the components could be zero for certain words, and for ambiguous words, x could have multiple values. In addition, there is an assumption of a unidimensional continuum along which every word must fall. The model also predicts, for example, that context effects will be independent of the word used and the individual involved. Although these and other simplifications reduce the model's explanatory and predictive power considerably, it was probably the first significant attempt made to quantify meaning.

More interesting than Mosier's (1941) theoretical formulations are his empirical investigations in the same article. He had subjects rate a list of evaluative adjectives (e.g., unsatisfactory, excellent) along a favorable-neutral-unfavorable continuum. He then scaled the responses by the method of successive intervals. The scale values for each word were interpreted as the constant component of the meaning of the word, while the spread of the distribution (i.e., the vagueness) was interpreted as ambiguity, or the variable component. The data tended to support his model in that it was possible to assign each word to a scale value along a unidimensional continuum, and the variation in meaning about this scale value was normally dis-

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tributed. (These results were later supported by Jones and Thurstone, 1955, for a set of preference words and phrases.)

Mosier's subjects also rated a subset of the adjectives paired with adverbial modifiers, or intensifiers (e.g., very, extremely). An analysis of these data showed that the addition of an intensifier caused a shift in the *meaning* of the base word away from the neutral point toward the extreme. This finding that adverbs tended to cause a shift in the scale values served as the basis for a later study on the influence of adverbs by Cliff (1959).

Mosier had shown that the meaning of an evaluative adjective could be represented as a point along a unidimensional continuum reflecting favorableness. Osgood and his colleagues (Osgood, 1952; Osgood, Suci, & Tannebaum, 1957) felt that it was possible to specify the meaning of a wider range of words by rating the words on a judiciously chosen set of scales. These scales were combined to form the semantic differential. Using this procedure, a subject is presented with a word (e.g., quicksand), which he characterizes by rating it along a number of antonymous scales, such as clean-dirty, fast-slow, and serious-humorous. The resulting profile is considered to be a multidimensional representation of the meaning of the word. Osgood's (1957) factor analysis of the various scales of the semantic differential yielded three main factors, which could be called *potency*, *activity*, and *evaluation*.

According to Osgood (1957), the similarity between two words (or between the same word for different populations) is interpreted as the Euclidean distance between corresponding profiles. Using this definition of the distance between concepts, Rowan (Note 1) had subjects rate a list of words (e.g., sleep, hero, gentleness) using the semantic differential and the method of triads. The distances between word pairs obtained from a multidimensional scaling of the triad data correlated highly with the distances from the semantic differential. In addition, the first two dimensions from the multidimensional scaling could be interpreted as evaluation and po-

tency/activity, the major factors derived from the semantic differential.

In the Mosier (1941) and the Jones and Thurstone (1955) studies, the rating scale (unfavorable-neutral-favorable) could be considered to capture a dimension of the meaning of the terms used. Thus, the words were rated according to the relative value of the corresponding denotative, or extensive, component. These ratings can be interpreted as the meaning of the words to the extent that both share a similar quantitative interpretation in the same context. Similarly, the semantic differential seems to demonstrate that subjects have the ability to rate the words in such a manner that overall differences between words are reflected in differential scale values. The semantic differential, however, does not demonstrate that the scale values reflect in any way the overall meaning of a word. Subjects may have the ability to rate a word such as *moon* as having a value on a *kind-cruel* scale. Even if this rating reflects a connotative property of the word, connotation is but one aspect of the meaning of the word. However, most scales on the semantic differential appear to reflect such connotative components. To suggest that word meaning can be uniquely specified by any number of such (nonorthogonal) scales implies that these scales represent all possible qualitative components in meaning—connotative and denotative. It has not been demonstrated that the semantic differential represents the full meaning of words in any comprehensive manner. Whether such a theory could ever adequately represent meaning is unclear.

The above studies have all represented meaning as a point along one or more rating scales. Mosier (1941) showed that the meaning of evaluative adjectives could be represented by scale values. How then might adverbs be characterized? Cliff (1959), extending Mosier's findings, proposed a formal model where adverbs functioned as multiplicative constants. If the scale value of the j th adjective is s_j , then the value of an adverb-adjective combination is represented as:

$$x_{ij} = c_i s_j + K,$$

where x_{ij} equals the obtained scale value of the i th adverb in combination with the j th adjective, c_i equals the multiplying value of the i th adverb, s_j equals the psychological scale position of the j th adjective, and K equals the difference between the arbitrary zero point of the obtained scale values and the psychological zero point of the scale.

To test the model, Cliff had subjects rate a set of adjective-adverb combinations (e.g., extremely lovable) along a favorable-unfavorable continuum, as in the Mosier study. Over all combinations of 10 adverbs and 15 adjectives, the predictions of the model were extremely accurate. Values of c_i and s_j were completely independent. The only deviation from the model was that the value of K appeared to vary slightly as a function of the adjective employed. Overall, the hypothesis that adverbs operate as multipliers was clearly confirmed. The results explain why combinations such as *unusually average* appear so awkward: The scale value of *average* is approximately zero, so multiplying zero by any finite value will leave a product of zero—the adverb is superfluous.

Quantification of Vagueness

An underlying assumption in the work on meaning quantification has been that the meanings of words can be specified as points along a scale. The variability about the scale value is attributable to the statistical nature of the system. Recent developments in semantic theory, such as the work of Labov (1973) and Lakoff (1973), have brought into question the assumption that meanings are precise. Instead, it has been proposed that natural language terms are to a lesser or greater degree inherently vague, such that, the boundary of a term is never a point but a region where the term gradually moves from being applicable to nonapplicable. Though this question of the inherent vagueness of language has only recently become of concern to psychologists, it has occupied the attention of philosophers since the time of the Greeks, who posed the question of vagueness in the form of the paradoxes of *sōrites* (the heap)

and *falakros* (the bald man). The latter paradox might take the form: How many hairs must be plucked from a man's head before he is considered to be bald? And given that there occurs a transition from being not bald to being bald, where is the one strand of hair that determines the transition, that is, where does one draw the line?

This paradox raises obvious difficulties for the law of the excluded middle: It appears unreasonable to assume that for a concept such as baldness every element of the universe is either a clear member of the set or a clear member of the complement of the set. The problem does not exist for artificial concepts with sharply defined boundaries, but appears unsolvable for vague, natural language concepts.

But what is a vague concept, and can it be specified? This was a rather productive area of research in philosophy during the early part of the twentieth century (e.g., Black, 1937; Copilowish, 1939; Hempel, 1939; Peirce, 1902; Russell, 1923; see also Black, 1963; Korner, 1957; Labov, 1973; Schmidt, 1974). Russell (1923) argued that the world is neither vague nor precise: It is what it is. Vagueness (and its complement, precision) are characteristics that can only belong to a symbolic representation, such as language. He further argued that a representation is vague when there exists a one-to-many relationship between the representing system and the system being represented.

Black (1937) agreed that the problem of vagueness is a property of natural language, but he faulted Russell for confounding vagueness and generality (p. 432, Note 12). Black differentiated these two terms from a third, ambiguity. Generality is what Russell described as vagueness, that is, a one-to-many relationship between a symbol and the items that the symbol represents. Ambiguity is the state of affairs where the same phonetic form has a finite number of alternative meanings. The vagueness of a symbol, however, will not be found as the result of a one-to-many relation nor a number of alternative meanings. Black reasoned that vagueness is a feature of the

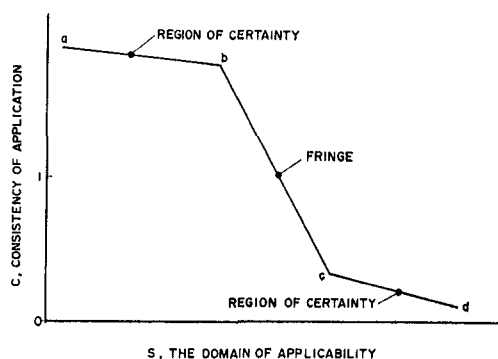


FIGURE 1. Hypothetical consistency profile.

boundary of a symbol's extension, not of the symbol itself. No matter how closely one looks or accurately one measures, the vagueness remains.

Peirce (1902) defined vagueness in a similar manner: "A proposition is vague when there are possible states of things concerning which it is intrinsically uncertain whether, had they been contemplated by the speaker, he would have regarded them as excluded or allowed by the proposition" (p. 748). There are some objects that a group of speakers of a language would definitely consider to be *chairs*; others that would never be called *chairs*. However, there will always be some objects that tend to straddle the boundary; no matter how closely one examines them, these objects cannot be classified as clearly belonging or not belonging to the category in question. It appears then that vagueness enters in the process of mapping a linguistic term onto a universe. That is, what is vague is the use of the linguistic term. (The term *natural language concept* thus refers to the result of this mapping operation, while *linguistic term* refers to the verbal label applied to the particular mapping. However, throughout the present article these two labels are used interchangeably to refer to both a term and its application.)

Black (1937), using what he termed a "consistency profile," first attempted a quantitative description of vagueness as defined by Peirce (1902). He hypothesized that while the vagueness of a word implies variability in the application of a term by

a group of language users, the variations should be specifiable and systematic. If they are not, it would be impossible to distinguish between terms. He defined the consistency of application of a term, T , to an element, s , of a set, S (i.e., $S = \{s\}$), as in:

$$C(T, s) = \lim_{\substack{M \rightarrow \infty \\ N \rightarrow \infty}} \frac{M}{N},$$

where M equals the number of judgments that T applies to s , and N equals the number of judgments that *not* T applies to s . The consistency profile was then defined as the function $C(T, s)$ over the domain of applicability, S . Figure 1 depicts a typical example, where it can be seen that the most doubtful cases correspond to $C(T, s) = 1.0$. This profile, then, was used to define the vagueness of a term by taking the slope of the curve from Point b to Point c (see Figure 1) as an index of vagueness.

In the same article, Black put forth alternative formulations for describing the vagueness of a term. He redefined the consistency of application as $T(s, C)$. Using this latter notation, a term, T , will be said to apply to an item, s , of a series, S , with a consistency, C . The term *not* T will then apply with a consistency, $1/C$. Thus, the law of the excluded middle can be replaced by a reciprocal relation where the product of the applicability of a term and its complement is always unity.

Attacking the same problem of vagueness in language, Hempel (1939) redefined the consistency of application of a term, T , to an object, s , as:

$$C(T, s) = \lim_{\substack{M \rightarrow \infty \\ N \rightarrow \infty}} \frac{M}{M + N},$$

where M and N are as defined above. As a consequence, the range of C is restricted to the closed interval 0 to 1, and the "doubtful" cases take on values of about 1/2. (In a recent study of word boundaries, Labov, 1973, implicitly used just this type of formulation to describe the manner in which the form of objects and context

interact to influence the extension of word boundaries.)

Hempel argued that Black's technique of specifying the vagueness of a term as the slope of the borderline region was inadequate, since the scale on the abscissa was arbitrary. Hempel reasoned, using his formulation, that the greater the vagueness in a term, the more objects there would be for which $C(T, s)$ takes on values around $1/2$. The precision (pr) of a term then could be formalized as:

$$pr = \frac{4}{N} \sum_{k=1}^N [C(T, s_k) - \frac{1}{2}]^2,$$

and the vagueness (vg) as:

$$vg = 1 - pr.$$

Specifying the vagueness of a term in this manner, independent of the slope of the consistency profile, avoids another problem encountered by Black (1937). As Figure 1 shows, there are two regions of certainty ($a - b$ and $c - d$) and a region connecting these, which has been called the borderline region, or fringe. Black specified the vagueness of a term as the slope of the fringe. But what delimits the fringe area? Are there precise points at which the uncertainty begins or ends (as depicted in Figure 1)? One seems drawn to the conclusion that the extent of the boundaries of a term's application seems to fade in and out in an almost imperceptible manner. There is a smooth, continuous transition from applicability to uncertainty to non-applicability. The entire extension of the term must then be considered to be vague, for the boundary extends (asymptotically) along the entire continuum.

The above discussion of the presence of vagueness in natural language argues against the Thurstonian model of meaning as put forward by Mosier (1941) and Cliff (1959). Such a theory implies that natural language concepts are precise and thus can be represented as points along a continuum. The observation that concepts form distributions is explained in terms of the statistical nature of the system. But if one

accepts the theoretical position that natural language concepts are inherently vague, then the point concept becomes an inappropriate theoretical construct. Not only can a vague concept refer to a range, but also the variability is an integral part of the meaning of the concept.

Theory of Fuzzy Sets

Computer scientists and systems engineers have long recognized that people can understand and operate upon vague, natural language concepts. Computers, however, are extremely rigid and precise information-processing systems. This inherent rigidity severely limits a computer's ability to abstract and generalize fundamental conceptual functions. Recently, Zadeh (1965, 1973) and others (e.g., Goguen, 1967, 1969; Santos, 1970; LeFaivre, Note 2) have developed quantitative techniques for dealing with vagueness in complex systems. The techniques are based on fuzzy set theory, a generalization of the traditional theory of sets.

The unique feature of fuzzy logic is that it allows complex systems to contain both numeric and linguistic variables, where a linguistic variable is defined as a label of a fuzzy set. For example, the linguistic variable *age* may take on values of *very young*, *rather young*, *young*, *middle age*, *not very old*, and so on, in addition to a range of numeric values. The assumption underlying these fuzzy sets is that the transition from membership to nonmembership is seldom a step function; rather, there is a gradual but specifiable change from membership to nonmembership. In nonfuzzy set theory a membership (characteristic) function specifies which elements are members of the set (i.e., for which elements $x \in X$ has a truth value of 1). In fuzzy systems, the grade of membership and the corresponding truth value of the proposition $x \in X$ may take on any value in the closed real interval from .0 to 1.0.

Fuzziness is distinctly different from uncertainty as measured by the probability of an event. The uncertainty of a coin toss resulting in a head has a certain probability

associated with it. No vagueness is involved—only lack of knowledge concerning an event occurring in the future. Once this knowledge becomes available, the state of affairs is completely determined. Referring back to the problem of *falakros* (the bald man), the concept *baldness* can be considered to be fuzzy. Unlike a coin toss, no matter how closely one measures or examines, the concept will apply more to some elements of the universe (men) than to others. No amount of information can make the boundary between *bald* and *not bald* free of imprecision.

To aid in discussions later in this article, it might be helpful at this point to summarize some of the important properties of fuzzy sets and show how traditional set theory is a special case of fuzzy set theory. (Much of this discussion is taken from Zadeh, 1965 and 1973.) Let X be a universe of points (or elements, or objects, or . . .) with a generic element of X denoted by x . A fuzzy subset of X , labeled A , is characterized by a membership function, f_A , that associates with each element x in X a real number, $f_A(x)$, in the closed interval from .0 to 1.0, which represents the grade of membership of x in A . An element of the fuzzy set A thus can be designated by the ordered pair:

$$[f_A(x), x], \quad (1)$$

where $f_A(x)$ is the grade of membership of x in A . Note that the nearer the value of $f_A(x)$ to 1.0, the higher the grade of membership of x in A . When A is a nonfuzzy set, its membership function can take on only values of 1 and 0 according to whether x does or does not belong to A , respectively.

As an example, consider the concept *tall*, where the membership function specifies the grade of membership of heights (in inches) in the set labeled *tall*.¹ Representative values might be: $f(60) = .0$, $f(66) = .2$, $f(68) = .7$, and $f(74) = 1.0$. Thus someone whose height is 5 ft. (152 cm) clearly is not tall, someone 5 ft. 8 in. (173 cm) is more tall than not tall, and someone 6 ft. 2 in. (188 cm) is clearly tall. It is important to realize that although the membership function might be defined precisely, it

does not follow that the concept itself is precise over the domain.

Membership in a fuzzy set (as in a nonfuzzy set) is specified by a mapping from the universe to the set in question. This mapping may be performed by enumeration, by a function, by an algorithm, and the like. Whatever the method, the result will be that every element in X will have associated with it a number corresponding to its grade of membership in that fuzzy set. Note that the relations among the elements (or among relevant dimensions of the elements) need not specify a continuum. Where the relevant relations among the elements cannot be ordered, the mapping onto the fuzzy subsets would be by enumeration or an algorithm. However, where at least an ordinal relationship of elements (along one or more relevant dimensions) is implied, the grade of membership may be determined by a function, as well as by enumeration or an algorithm. Once this mapping is specified, the set can be used as a linguistic variable in fuzzy inferences and algorithms and can be modified by operations such as negation and union.

In both fuzzy and nonfuzzy set theory the *support* of the set A is the set of elements in X for which $f_A(x) > 0$. A (fuzzy or nonfuzzy) *singleton* is a set whose support is a single element in X . Therefore, a fuzzy set A may be specified as the union (see Equation 10) of its constituent singletons. Thus A may be represented by:

$$A = \int_X [f_A(x), x], \quad (2)$$

¹For directness of exposition, concepts are assumed to be context constant (but not context free) here and throughout the study. In terms of membership functions, this implies that although the functions most likely contain more than one independent variable, the context-relevant variables have been held constant. For example, the membership function for *tall* is obviously influenced by the perspective (i.e., height) of the respondent. Assuming a constant perspective here in no way limits the applicability of the fuzzy set approach. (But see Labov, 1973, for a preliminary investigation of the interaction of form and context in referential meaning.)

where the integral sign represents the union of the fuzzy singletons $[f_A(x), x]$. If the fuzzy set A has finite support (i.e., if there are a finite number of elements whose grades of membership are greater than 0), then Equation 2 may be replaced by the summation:

$$A = [f_A(x_1), x_1] + [f_A(x_2), x_2] + \dots + [f_A(x_n), x_n], \quad (3)$$

or

$$A = \sum_{i=1}^n [f_A(x_i), x_i], \quad (4)$$

in which $f_A(x_i)$ is the grade of membership of x_i in A . Note that the $+$ sign in Equation 3 specifies the union of elements (see Equation 10) and not the algebraic sum. For example, given that the domain is the natural numbers, a fuzzy set labeled *few* might be defined as:

$$\begin{aligned} few = & (.2, 1) + (.2, 2) + (.8, 3) \\ & + (1.0, 4) + (1.0, 5) \\ & + (.8, 6) + (.5, 7), \end{aligned} \quad (5)$$

where the support of *few* is the set of numbers 1, 2, . . . , 7; that is, according to the definition, numbers greater than 7 have a grade of membership of .0 in the set labeled *few*. Notice that each ordered pair relates a natural number to the grade of membership of that number in the concept. This relationship may be extended by having the grade of membership itself be a fuzzy set. For example (from Zadeh, 1973), if the universe, X , is:

$$X = \text{Tom} + \text{Jim} + \text{Dick} + \text{Bob}, \quad (6)$$

and A is the fuzzy set *agile*, then:

$$\begin{aligned} agile = & (\text{medium}, \text{Tom}) + (\text{low}, \text{Jim}) \\ & + (\text{low}, \text{Dick}) + (\text{high}, \text{Bob}). \end{aligned} \quad (7)$$

Low, in turn, is a fuzzy subset of the universe of possible grade of membership values (referring mainly to values near .0), or:

$$\begin{aligned} low = & (.5, .2) + (.7, .3) + (1.0, .4) \\ & + (.7, .5) + (.5, .6). \end{aligned} \quad (8)$$

A *crossover point* in A is defined as an element that possesses a grade of membership in A of .5. Thus, *penguin* might be a crossover point in the set of birds, meaning that a penguin is as much a bird as it is not a bird.

Two fuzzy sets, A and B , are *equal* ($A = B$), if and only if $f_A(x) = f_B(x)$ for all x in X . Where the grades of membership take on values of either 0 or 1, this relation reduces to the traditional set theory definition of the equality of two sets. A is *contained* in B (or B *entails* A , or A is a fuzzy subset of B , i.e., $A \subset B$) if and only if $f_A(x) \leq f_B(x)$ for all x in X .

Operations on Fuzzy Sets

Negation, logical and algebraic operations, hedges, and other terms that influence the representation of linguistic variables can be considered as labels of various operations defined on fuzzy subsets of X . The more basic operations will be reviewed here. For a more complete overview, see Zadeh (1965, 1973) and Goguen (1967, 1969).

The *complement* of a set A is denoted *not* A , or \bar{A} , and is defined as:

$$\text{not } A = \int_X [1 - f_A(x), x], \quad (9)$$

that is, as in nonfuzzy set theory, the operation of complementation corresponds to negation. It is interesting to note that Black's (1937) reciprocal model of negation can be shown to be isomorphic to this complement operation.

The *union* of two fuzzy sets, A and B , is denoted $A + B$, and is defined as:

$$A + B = \int_X [f_A(x) \wedge f_B(x), x], \quad (10)$$

where

$$f_A(x) \wedge f_B(x) = \max[f_A(x), f_B(x)]. \quad (11)$$

The union of two sets corresponds to the logical *or* operation. In fact, in the situation where the grades of membership take on values of only 0 and 1, Equation 10 reduces to the Boolean *or* operation of set theory.

TABLE 1
LABELS OF FUZZY SETS AND THEIR
PREDICTED OPERATIONS

Label	Operation
SMALL	f_{small}
LARGE	f_{large}
NOT SMALL	$1 - f_{small}$
NOT LARGE	$1 - f_{large}$
VERY SMALL	f_{small}^2
VERY LARGE	f_{large}^2
NOT VERY SMALL	$1 - f_{small}^2$
NOT VERY LARGE	$1 - f_{large}^2$
VERY VERY SMALL	$f_{small}^4 = f_{very\ small}^2$
VERY VERY LARGE	$f_{large}^4 = f_{very\ large}^2$
NOT VERY VERY SMALL	$1 - f_{small}^4$
NOT VERY VERY LARGE	$1 - f_{large}^4$
EITHER LARGE OR SMALL	$\max[f_{large}, f_{small}]$

The *intersection* of two fuzzy sets is denoted $A \cap B$ and is defined as:

$$A \cap B = \int_x [f_A(x) \wedge f_B(x), x], \tag{12}$$

where

$$f_A(x) \wedge f_B(x) = \min[f_A(x), f_B(x)]. \tag{13}$$

The intersection corresponds to the logical connective *and*, which also reduces to the Boolean operator for grades of membership of 0 and 1.

The above descriptions serve to demonstrate how various linguistic operators can be defined in terms of fuzzy sets. Zadeh (1972) and Lakoff (1973) have attempted to demonstrate that various linguistic operators (e.g., *very*, *rather*) can likewise be incorporated into the system of fuzzy logic by being considered as additional operators upon linguistic variables. For example, the adverb *very* appears to act as an intensifier. Zadeh reasoned that given a fuzzy set labeled A , *very A* should be of the form:

$$very\ A = \int_x [f_A^2(x), x]. \tag{14}$$

This formulation was later generalized to:

$$very\ A = \int_x [f_A^a(x), x],$$

where $a > 1.0$. (15)

Both Lakoff and Zadeh further analyzed

operations that were concatenations of the primitive operations. For example, given the definitions of *not* (Equation 9) and *very* (Equation 14), the operation of *not very A* implies:

$$not\ very\ A = \int_x [1 - f_A^2(x), x]. \tag{16}$$

As part of the axiomatic system of fuzzy set theory, Zadeh (1968) generalized to fuzzy set theory the relation between the probability (P) of an event A , and the expected value of its membership function, that is:

$$P(A) = E(f_A). \tag{17}$$

Equation 17 has certain direct psychological implications. For example, the application of a term to an element might imply a grade of membership between .0 and 1.0. However, the overt judgment of the relation of the term and element might require a binary (yes-no) reply. Using Equation 17, the distribution of binary responses can be related to the grade of membership of the element in the fuzzy set labeled by the term. A technique is thus available for empirically verifying the predictions of fuzzy set theory in terms of the implications for human information processing.

Psychological Implications

Zadeh (1973) proposed that in dealing with humanistic systems we apply the

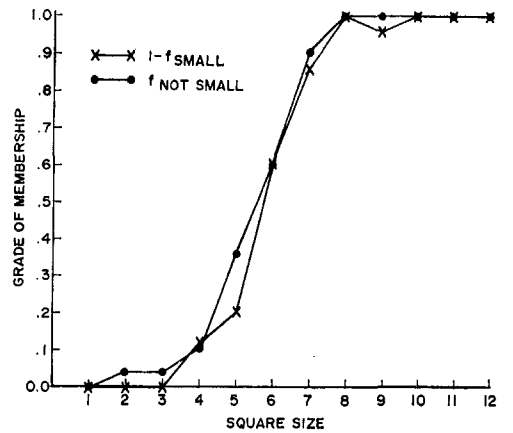


FIGURE 2. Membership function for *not small*.

principle of incompatibility: "As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics" (p. 28). Given the complexity of the system that psychologists study, one need not search far in order to appreciate the applicability of the above principle to the study of behavior.

This principle can actually be applied from two separate viewpoints. The first is in the description and explanation of behavior and in data analytic techniques. Some work has already been started in such areas as role theory (Thomson & Marinos, 1972), pattern classification (Bellman, Kalaba, & Zadeh, 1966), and clustering techniques (Ruspini, 1970; Bezdek, Note 3). The second view is in describing and explaining how people interact with the world around them. While both lines of research could profit greatly from utilizing these new quantitative techniques provided by fuzzy set theory, we have chosen the latter line of inquiry.

It was shown in the section on vagueness that people have the ability to comprehend and manipulate vague concepts. If this behavior could be described in terms of fuzzy logic, this new quantitative technique might motivate the development of new methods for the modeling of language

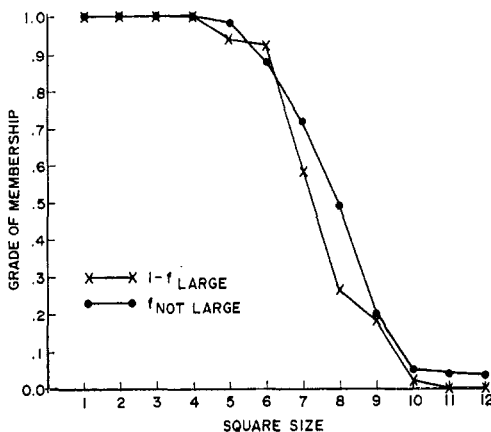


FIGURE 3. Membership function for *not large*.

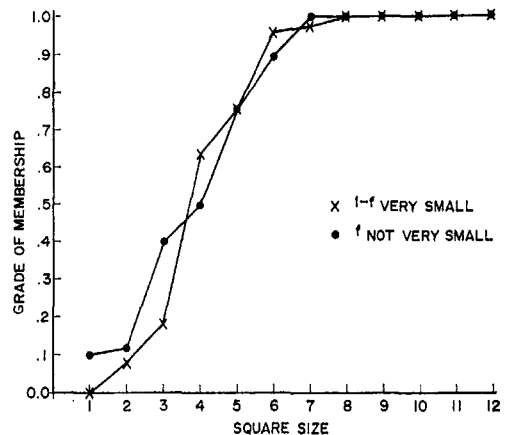


FIGURE 4. Membership function for *not very small*.

comprehension, reasoning, natural problem solving, and other complex behavioral processes. To be more specific, the theoretical position that runs through this line of research is that people comprehend vague concepts (i.e., all natural language concepts) *as if* the concepts are represented as fuzzy sets. Moreover, people manipulate vague concepts *as if* they are processing according to the rules of fuzzy logic. Several preliminary studies appear to support this theoretical position.

EXPERIMENT 1

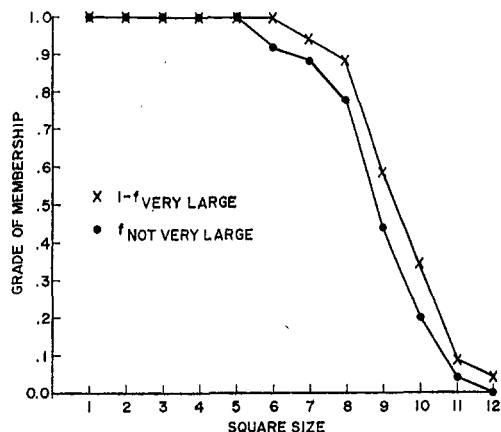
Since Zadeh (1972) and Lakoff (1973) both make explicit claims as to the manner in which fuzzy sets should be transformed, it was felt that a logical first step would be to obtain some baseline fuzzy sets and empirically evaluate the transformations that occur when these sets are operated upon by negation and the addition of hedges.

Method

Subjects. Nineteen undergraduates at The Johns Hopkins University served as paid subjects.

Stimuli. Twelve slides, each containing a black square on a white background, were used as the physical stimuli. When projected on the screen, the squares measured 4, 6, 8, 10, 12, 16, 20, 24, 28, 32, 40, and 48 in. (10.2 cm to 121.9 cm) on one side.

Labels of the fuzzy sets consisted of the adjectives *large* and *small* paired with various combinations of *not* and the intensifier *very*. In addition, in order to test the concept of fuzzy union, the phrase

FIGURE 5. Membership function for *not very large*.

either large or small was also used. The 13 labels are shown in Table 1, along with the predicted transformations from Zadeh (1973).

Procedure. Subjects were run in one group. Each received an answer sheet containing the 13 phrases in one of 10 random orders. Below each phrase were 12 spaces. Subjects were instructed to look at the first phrase. They were told that they would be shown 12 squares in a random order. They were to simply look at each square and decide whether the phrase applied to it. If it was appropriate, they were to enter *yes* in the appropriate space; if it was not appropriate, they were to enter *no*. This procedure was repeated for each phrase, with a different random order of squares in each block. Slides were exposed for 1 sec. Subjects were given additionally 5 sec in which to respond. Before the start of the experiment the 12 squares were shown in ascending order to insure that everyone was operating within approximately the same context.

After every block of 12 slides, subjects were given a 30-sec break. After all 13 blocks, they were given an additional 5-min. rest, and the process then was repeated with different randomizations. Thus every subject responded twice to every combination of square and phrase. The entire session lasted 50 min.

Results

Since Zadeh (1968) had equated the probability of a fuzzy event with the expected grade of membership of the event, the proportion of *yes* responses for a particular square and phrase was interpreted as the grade of membership for that square in the fuzzy set labeled by the phrase. (The data were scaled by the method of successive intervals [Diederich, Messick, & Tucker, 1957] to obtain a

psychological scale for the square size. Since the use of this scale in no way changed any of the results, the data are reported simply in terms of the ordinal square size.)

Figures 2 and 3 show the membership functions for *not small* and *not large*, respectively. In each graph the negative phrase is plotted with the complement of the corresponding affirmative phrase. In these figures, as in all remaining figures, the ordinate corresponds to the grade of membership (truth value), while the abscissa corresponds to the ordinal square size. Figures 4 and 5 relate *not very small* to the complement of *very small* and *not very large* to the complement of *very large*, respectively. Figures 6 and 7 show the corresponding relations for *very very small* and *very very large*, respectively. The fact that the graphs indicate a reasonably good fit, and the fact that the average root mean square error over all positive-negative pairs was less than .07 supports the fuzzy set notion of negation as being the complement of the positive set.²

² Note that these results can likewise be predicted by a Thurstonian model where the concept is assumed to be constant and the evaluation of the square size is the variable component. Fuzzy set theory assumes that the response variability is mainly due to the vagueness of the verbal concept: The perception of square size is relatively constant. Differences at this point are more a matter of theoretical perspective than of quantitative prediction.

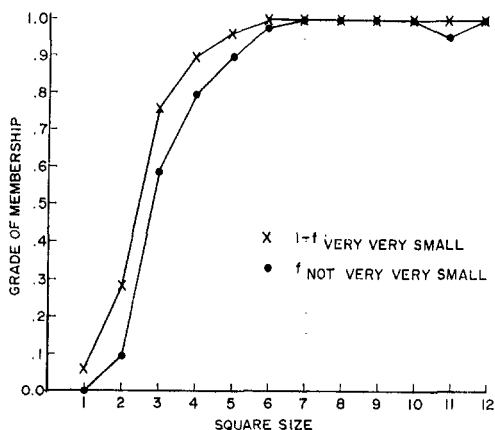
FIGURE 6. Membership function for *not very very small*.

Figure 8 is a graph of *not large* and *small*. Although the two functions are monotonically decreasing, they do not represent the same concept. *Not large* appears to extend the concept *small* to include the midrange of the continuum.

The effect of the intensifier *very* on the concepts *small* and *large* are plotted in Figures 9 and 10, respectively. Note that the relation between *large* and *very large*, *very large* and *very very large*, and so on corresponds to the definition of entailment for fuzzy sets. Within each triplet (e.g., *small*, *very small*, *very very small*) the slopes of the functions appear approximately equal. (This was confirmed by the successive intervals analysis.) The equality of the slopes implies that the influence of *very* did not appear to reduce the vagueness of the concept. Now if Zadeh's (1972, 1973) hypothesis concerning the functioning of *very* as a power function is accurate, the slope of the function should increase as the intensifiers are concatenated. This does not seem to be the case.

Various classes of operators were tried in order to find a reasonable mapping function from a concept to the concept modified by *very*. Although a reasonable fit was obtained with a power function, it was felt that a more straightforward explanation was appropriate. In examining the plots of the membership functions it appeared that the addition of the intensifier

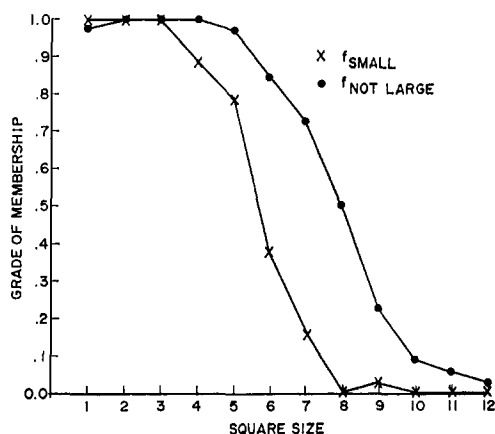


FIGURE 8. Membership functions for *not large* and *small*.

very served to simply translate the function along the abscissa.³ Support for this finding comes from the work of Cliff (1959), who found that adverbs tended to function as multiplicative constants. (Recall that Cliff was working under the assumption that the meaning of an adjective could be characterized as a point on a scale: The adverb functioned to shift the point along the scale.) A least squares method was used to obtain the best shift for *very* and *very very*. However, for ease of exposition, it is sufficient to describe the translation for *very* as two ordinal scale units and *very very* as an additional ordinal scale unit toward the extreme of the scale.⁴ The results of this shifting operation are shown in Figures 11 and 12.

Figure 13 is a plot of the phrase *either large or small* and the operation of fuzzy union on the concepts *large* and *small*. While there is a definite similarity in the shape of the two functions, the discrepancies are more than minimal. The plot of the phrase is depressed in relation to the

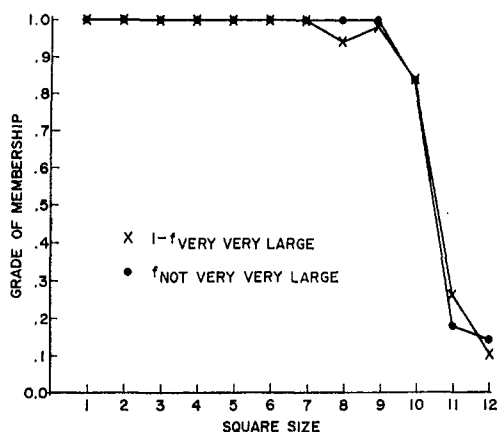


FIGURE 7. Membership function for *not very very large*.

³ Root mean square errors overall were .074, .214, and .061 for the best-fitting power, exponential, and translation functions, respectively.

⁴ From the successive intervals scaling of the psychological square size, the ratio of the best translation to the mean intersquare distance was actually 1.81 and 1.15 for *very* and *very very*, respectively.

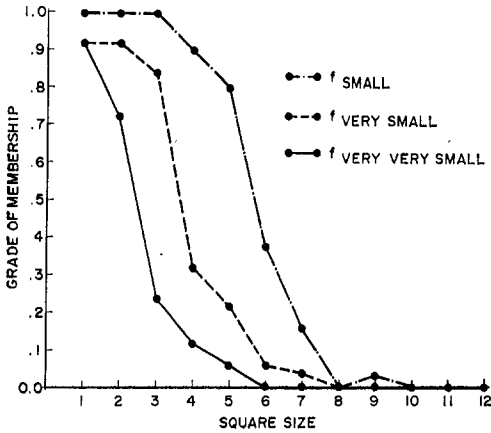


FIGURE 9. The effect of *very* on the concept *small*.

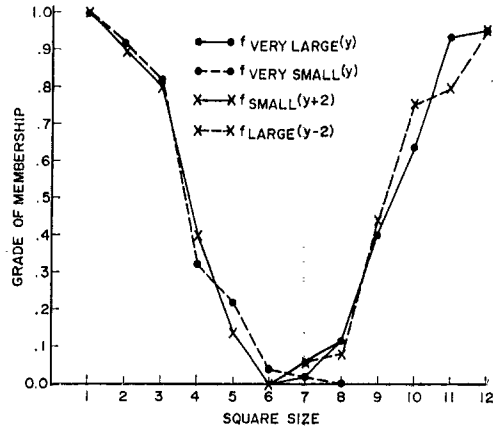


FIGURE 11. *Very* as a translation operation.

fuzzy set theoretic prediction. Some of the discrepancy is probably due to the difficulty that some subjects reported in evaluating this concept: the result being that the data contains possible inverted responses.

Overall, several conclusions are clear. Concepts defined by extension can be considered as fuzzy sets, and people tend to interpret these concepts and their variations in terms of operations on fuzzy sets. Negation can be characterized by the complement of the positive set. The effect of the intensifier *very* can also be interpreted as an operation on a fuzzy set, although its empirical form deviates from the predicted function. Operators like *not* and *very* can be concatenated with themselves

and each other, with predictable results.

It would have been nicer if Zadeh's (1972, 1973) hypothesis of *very* operating as a power function of the grade of membership was applicable. Although such a finding would have supported the generality of the operation of *very*, the form of the operation is an empirical question. *Very* defined as a translation operation is obviously class dependent: It can certainly be generalized to other relative adjectives (e.g., *good*, *short*, *hot*). How such an intensifier would operate on the class of absolute adjectives (e.g., he is *very* British, this is *very* red) remains to be determined. Intuitively, it appears that the meaning of *very* in *very* large is qualitatively different from *very* in *very* British. The former implies an extreme of a continuum; the latter implies a greater emphasis on characteristic features (Lakoff, 1973).

The most important point of Experiment 1 is that no longer is the variability about the mean of a distribution considered to be strictly the result of noise in the system or the uncontrollable influence of extraneous variables. The variability has a specific interpretation in terms of the grade of membership of the element in the fuzzy set denoted by the concept label.

EXPERIMENT 2

Experiment 1 demonstrated that the composite responses of a group of subjects

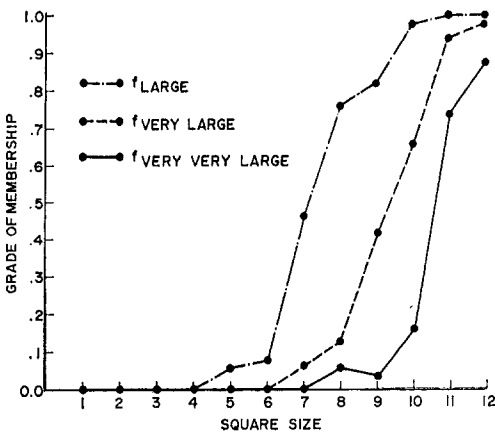
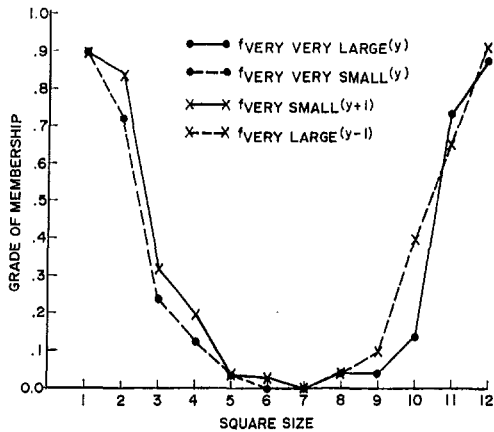


FIGURE 10. The effect of *very* on the concept *large*.

FIGURE 12. *Very very* as a translation operation.

could be considered as reflecting operations on fuzzy sets. The argument would be more convincing if it could be shown that individual subjects display similar behavior. Experiment 2 therefore can be considered a replication of Experiment 1 with repetitions across days for a single subject rather than across subjects for a single presentation.

Method

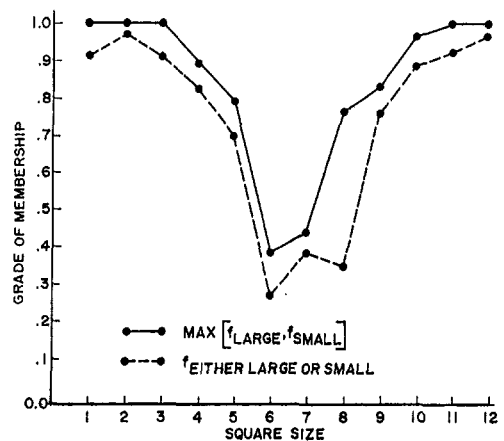
Subjects. Four students at The Johns Hopkins University, who had not been in Experiment 1, served as paid subjects.

Procedure. The experimental procedure was similar to Experiment 1. Exposure time for the slides remained at 1 sec; however, the intertrial intervals were under subject control. In order to obtain more information from each trial, confidence ratings were also used. Subjects were instructed to respond *yes* or *no* according to whether the square viewed was appropriate for the phrase. After this response was recorded, subjects rated how confident they were in their decision on a 5-point scale, where 1 was interpreted as purely guessing and 5 as absolutely certain.

In a typical session, which lasted less than 1 hr., a subject judged all combinations of the 12 squares and 13 phrases twice. Each subject participated in five such sessions over 5 days.

Results

The binary decisions and the confidence ratings were integrated into a single scale over the closed interval .0 to 1.0. For example, a *no* judgment with a confidence rating of five was assigned .0, while a *yes* response with a similar confidence rating

FIGURE 13. Membership function for *either large or small*.

was assigned 1.0.⁵ Although there is no clear theoretical justification for this mapping, the resulting scale seemed to reflect the important conceptual ideas implicit in fuzzy set theory. The extremes of definite membership and nonmembership correspond to judgments that were rated as highly confident. Likewise, judgments rated

⁵ More formally, if d is the binary applicability decision (1 = yes, -1 = no) and r is the value on the confidence scale ($1 \leq r \leq 5$), then the grade of membership is defined as:

$$\text{Grade} = 5. + d\left(\frac{r}{10}\right).$$

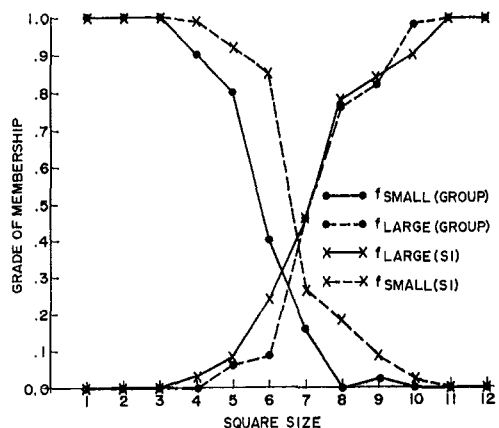


FIGURE 14. Membership functions from Subject 1 and the group data.

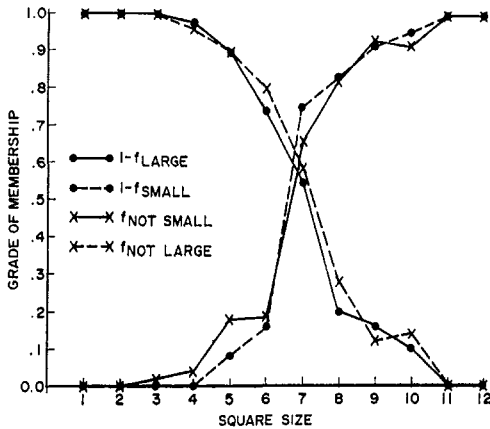


FIGURE 15. Membership functions from Subject 1 for *not large* and *not small*.

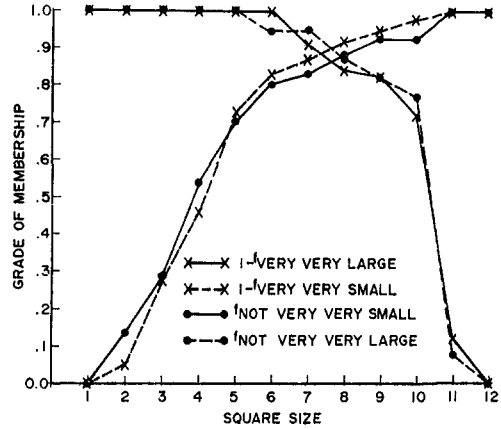


FIGURE 17. Membership functions from Subject 1 for *not very very large* and *not very very small*.

as purely guessing corresponded to the crossover points in a fuzzy set.⁶

The results of three of the four subjects resembled, with minor differences, the results of the group experiment (Experiment 1). Moreover, since there were no important differences among Subjects 1-3, only Subject 1 will be discussed. Subject 4 was quite different and will be discussed separately.

The results of Subject 1 are shown in Figures 14-20. Figure 14 compares the membership functions for *large* and *small* with similar functions obtained from the group experiment (Experiment 1). Al-

though the functions for *large* appear very similar, the concept *small* is more extensive for Subject 1 than for the group. Information of this type might be useful in studies on individual differences in comprehension, but is irrelevant for the theory being considered. The hypothesis being tested is that the operations on fuzzy sets should remain constant, independent of the basic membership functions.

Figures 15, 16, and 17 compare the negative concept to the complement of the corresponding positive concept. As with Experiment 1, the functions are quite similar, implying that the effect of negating a concept is to replace the grades of membership with their complements.

Plots of *very* as a shifting operation are shown in Figures 18 and 19. As with the group data, the goodness of fit indicates that the translation operation is a tenable description of the effect of modifying a concept by *very*. Finally, Figure 20 shows that the membership function for *either large or small* shows a slight depression in relation to the fuzzy union of *large* and *small*, as was found in Experiment 1.

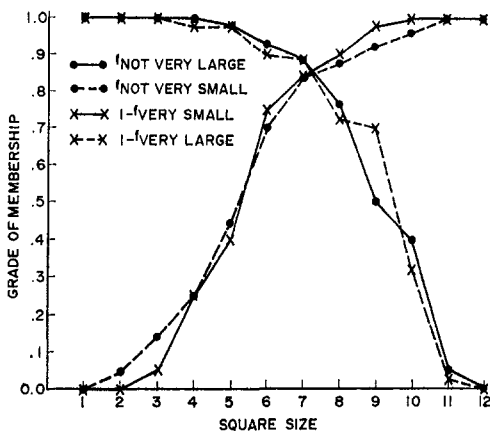


FIGURE 16. Membership functions from Subject 1 for *not very large* and *not very small*.

⁶ Inherent in this transformation is the assumption that equal confidence intervals reflect equal differential grades of membership. The comparison of the concept *large* for the group and for the individual subject, here Subject 1 (see Figure 14), tends to support this assumption.

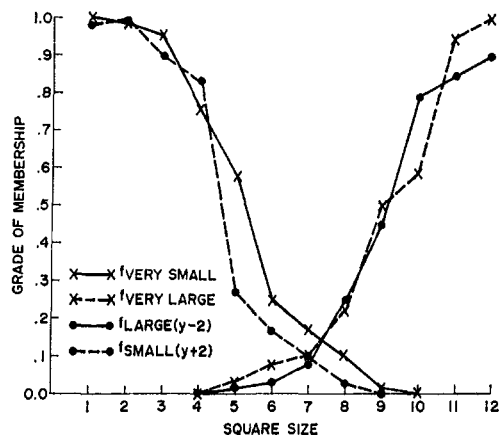


FIGURE 18. *Very* as a translation operation.
(Data are from Subject 1.)

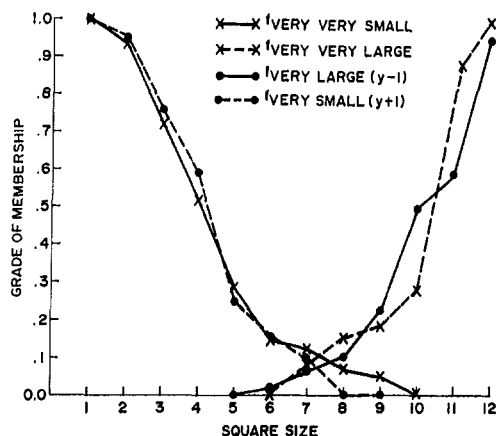


FIGURE 19. *Very very* as a translation operation.
(Data are from Subject 1.)

Of the 3 individual subjects and the 19 group subjects tested, all responded in a similar manner. Their judgments all reflected fuzzy logical operations where the concept appeared to dichotomize the continuum (in a fuzzy manner) into a region of inclusion and a region of exclusion. Although these findings do reflect the assumptions of fuzzy set theory—from the perspective of linguistic processing—some of the results are rather anomalous. According to Bolinger (1972), the use of litotes such as *not very* does not imply the complement of the continuum as Figures 4 and 5 show, but only some range near the middle of the continuum: For example, to say that someone is *not very tall* is typically interpreted as meaning that he is *rather short* or *sort of short*. Subject 4 made judgments drastically different from the other subjects and seemed to be responding according to this more “idiomatic” interpretation of the phrases.

Figures 21 and 22 show the various positive membership functions as perceived by Subject 4. Whereas the other subjects perceived the operation of *very* on the concept *small* as defining successively smaller fuzzy subsets, Subject 4 interpreted the phrases as (fuzzy) overlapping categories. A square that is judged to have a maximum grade of membership in the set *very very small* is considered to be a marginal ($f = .56$) member of the set *small*. From

the shape of the membership functions in Figures 21 and 22 it can be argued that the concept of strict entailment was not functioning for this subject. The fact that a square is judged to be *very very large* does not entail that it is also *large*.

The membership functions for *not small* and *not large* are presented in Figures 23 and 24, respectively, along with *large* and *small*. It appears that the two negative terms are rather general concepts representing the middle region of the contextual range. Functions for *not very small* and *not very large* are plotted in Figures 25 and 26, respectively. The functions seem to indicate that for this one subject the term *not* operates upon the intensifier *very* rather

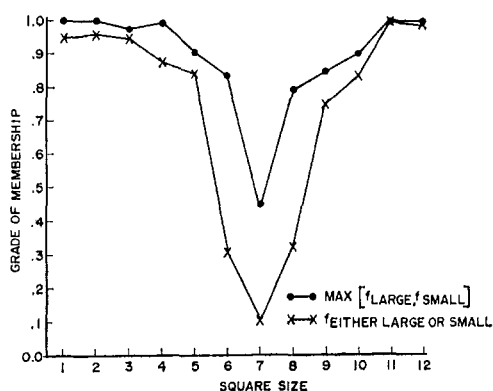


FIGURE 20. Membership function from Subject 1 for
either large or small.

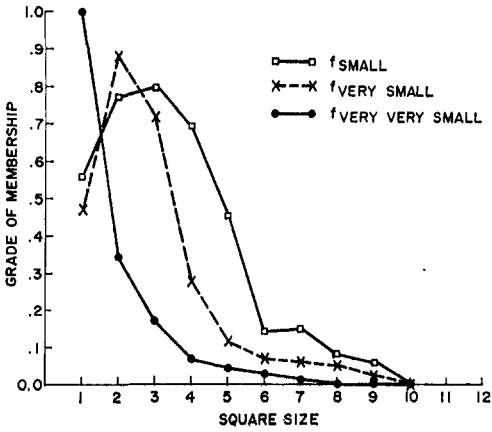


FIGURE 21. Membership functions containing *small* for Subject 4.

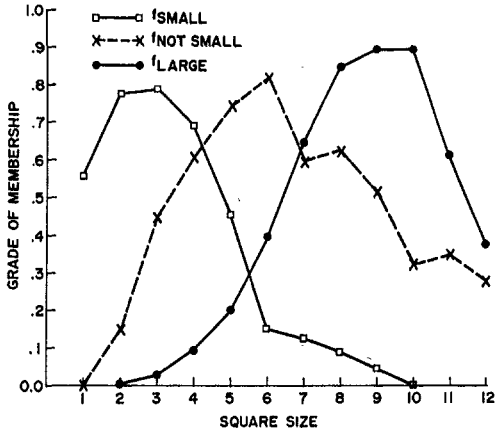


FIGURE 23. Membership function from Subject 4 for *not small*.

than on the entire concept. More explicitly, the original empirical formulation for *not very small* is:

$$f_{not\ very\ small}(x) = 1 - f_{very\ small}(x) \tag{18}$$
$$= 1 - f_{small}(x + d), \tag{19}$$

where d is the amount of translation on the abscissa. Over at least a portion of the domain, a tentative hypothesis for the behavior of Subject 4 might be:

$$f_{not\ very\ small}(x) = f_{very\ small}(x - 2d)$$
$$= f_{small}(x - d), \tag{20}$$

that is, the function of *not* operates on the

function *very* by translating the function about the base membership function. This formulation is extremely speculative, and the data to support the operation comes from only one subject out of 23. Nevertheless, it must be recognized that the empirical functions produced by this one subject, far from being random, appear to obey lawful relationships.

EXPERIMENT 3

The question remained as to why 22 subjects would respond in a (fuzzy) logically predictable manner, and only one subject respond in a manner that somewhat reflected a linguistic interpretation. Perhaps the demands of Experiment 2 were such as to force Subject 4 into responding in the more logical manner. To test this hypothesis Experiment 3 was performed; the context was varied in the hope that a more linguistic interpretation would result.

Method

Subjects. Fourteen undergraduates at The Johns Hopkins University, not involved in the previous experiments, participated as paid subjects.

Stimuli. The 12 squares used in Experiments 1 and 2 served as the physical stimuli. The terms were composed of the adjectives *large* and *small*, which appeared unmodified and in combination with *not*, *very*, and *not very*. In addition, the hedge sort of was included in order to evaluate the relation between *not very* and this relative adverb.

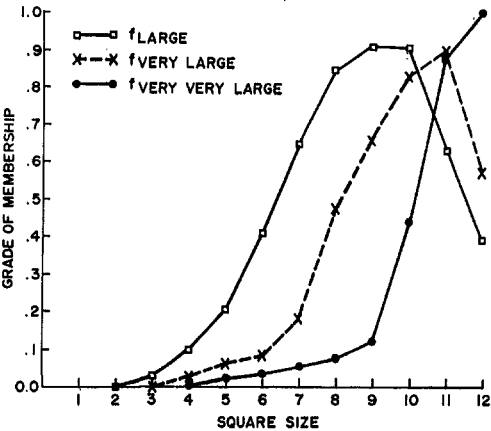


FIGURE 22. Membership functions containing *large* for Subject 4.

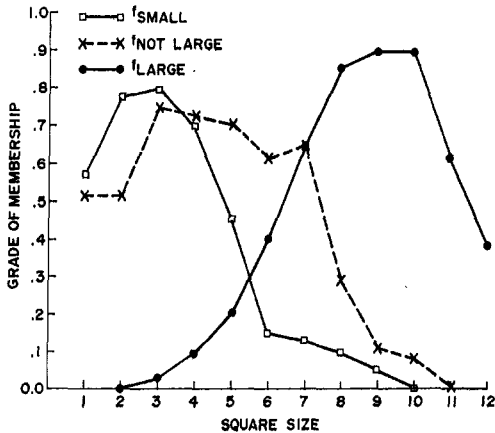


FIGURE 24. Membership function from Subject 4 for *not large*.

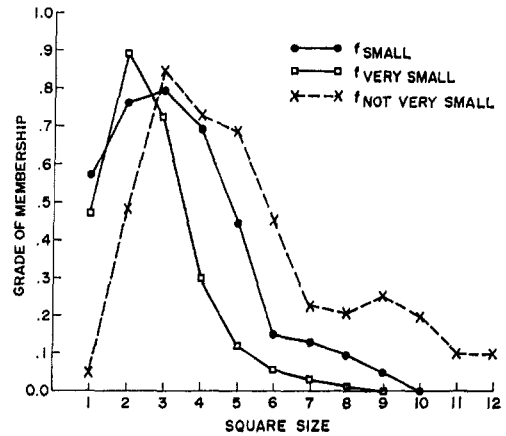


FIGURE 25. Membership function from Subject 4 for *not very small*.

The terms were integrated into sentences of the form, "This is sort of small," which were then recorded in a random order onto one track of a stereo magnetic tape. The hope was that the auditory presentation of the sentences might be reflected in a more natural language interpretation. Two identical tapes were made. On Tape 1 the phrases *not very large* and *not very small* received strong emphasis on the word *not*. On Tape 2 the word *very* was emphasized. The remaining sentences were dubbed from the same master tape and thus were identical. On the second track of each stereotape synchronization information to automatically advance the slide projector was recorded.

Procedure. Subjects were randomly assigned to one of two treatment groups corresponding to which audiotape they would hear. The procedure for each group was identical. On each trial a slide would be projected. At the same instant that the slide was exposed, a sentence would be heard. The subject was to simply look at the slide and decide whether the sentence he was hearing was appropriate with reference to the square on the slide. Subjects circled the appropriate response (*yes* or *no*) on an answer sheet and indicated on a rating scale from 1 to 3 their degree of confidence in their decision. After 10 stimulus pairs there was a 15-sec pause, and after every 60 trials a 2-min. rest period. Each subject saw all combinations of the 12 squares and 10 sentences two times in a completely randomized order. Before they began, subjects were shown the entire range of slides in ascending order and the range of sentences, so they would all be operating in approximately the same context. The entire session lasted less than 45 min.

Results

The findings of Experiment 3 demonstrate how robust the various concepts are under differing experimental conditions.

Figures 27 and 28 show, respectively, the correspondence between *small* and *large* from Experiments 1 and 3. It is apparent that neither the change in modality (visual/auditory), emphasis, context (phrases/sentences), nor design (blocked/completely randomized) had any influence on the results. The functions for the other concepts correspond in a similar manner. The only other result of interest is the membership functions for *sort of small* and *sort of large*. These functions are plotted in Figures 29 and 30, respectively, and compare favorably with the plots for *not very small* and *not very large* for Subject 4.

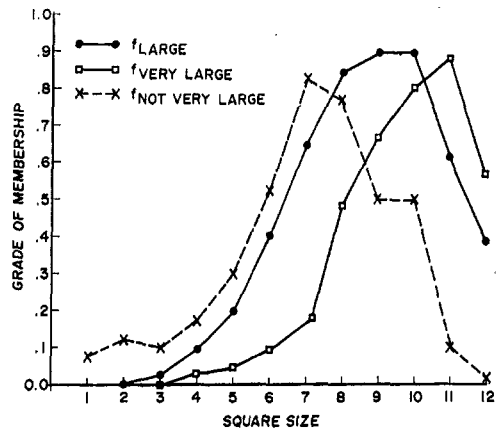


FIGURE 26. Membership function from Subject 4 for *not very large*.

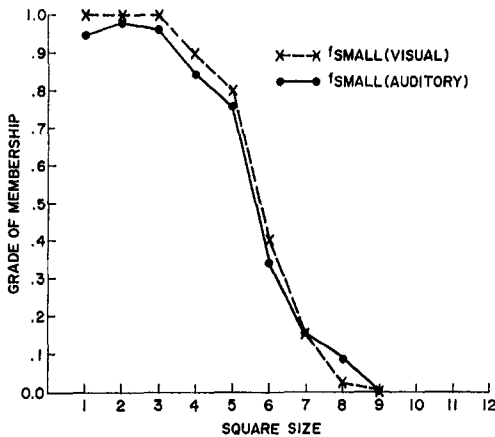


FIGURE 27. Membership functions for *small* from Experiment 1 (visual stimuli) and from Experiment 3 (auditory stimuli).

EXPERIMENT 4

In Experiments 1-3, having to make judgments on more than one phrase may have led subjects to treat concepts and modifiers in the observed "logically" predictable manner. This may result from subjects redefining the phrases in relation to each other. That is, the subjects may adopt a response strategy for specific phrases that is influenced by the other phrases used in the experiment. So, for example, the type of response to *not very small* may be a function of how the subject responded to *very small*. To overcome this possible contextual effect, an experiment was performed where each subject had to judge only a single phrase. It was felt that in this case, subjects might give a *linguistic* as opposed to *logical* interpretation to the phrases.

Method

Subjects. Forty-five undergraduates at The Johns Hopkins University, who had not participated in the earlier experiments, served as unpaid subjects. There were 10, 11, 12 and 13 subjects in the *large*, *very large*, *not very large*, and *sort of large* conditions, respectively.

Stimuli. Twelve black squares, proportional in size to the squares used in Experiments 1-3, were mounted on a white background. The mounted squares were fastened in ascending order (left to right) to a blackboard in the front of a classroom. Below each square was a number corresponding to

the ordinal ranking. Unlike the preceding experiments, only four phrases were used: *large*, *very large*, *not very large*, and *sort of large*.

Procedure. Subjects were given a sheet of paper containing a single phrase and were instructed to look at the 12 squares and write down the identifying number or numbers of squares that might correctly be characterized by the phrase written on their sheet. Experiment 4 lasted approximately 2 min.

Results and Discussion

Figure 31 shows the membership functions for the four phrases. Two results are important. First, there is a small but predictable decrease in the grade of membership of the largest square for the concept *large*. This result is consistent with a *linguistic* interpretation of *large*, where *large* is not strictly entailed by *very large*. Second, the phrase *not very large* is clearly not the complement of *very large*. Rather as predicted by the *linguistic* interpretation, it seems to be functioning as *sort of small* (compare Figures 30 and 31). What is important here is simply that *not very large* is not the complement of *very large*. These two results together suggest that subjects will tend to give *linguistic* interpretations to the phrases if experimentally induced context effects are eliminated. The question arises whether these results are inconsistent with the formalism of fuzzy set theory. We think not. At worst they will complicate the particular characterization of various

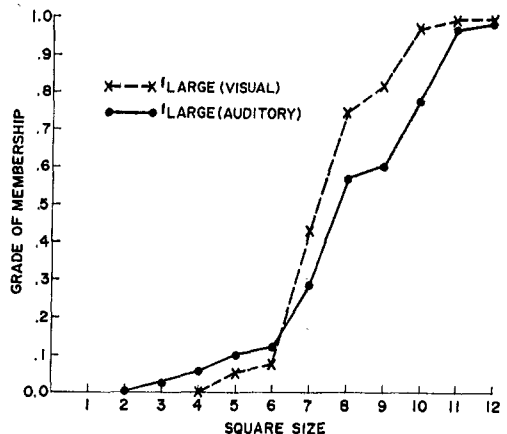


FIGURE 28. Membership functions for *large* from Experiment 1 (visual stimuli) and from Experiment 3 (auditory stimuli).

operators but not invalidate the principle claim that intensifiers and hedges can be described as operators on fuzzy sets. In fact, recall that we did give a formal, though tentative, formulation of the operations underlying the use of adverbs for the subject (Subject 4) who used a *linguistic* interpretation in judging the application of phrases to square size. The actual form that the operators and concepts take is an empirical issue to be determined by experimentation. Once we have determined the forms they take we can then proceed to see whether we can give them an interpretation within the framework of fuzzy set theory. This is essentially the procedure followed in the "odd" case of Subject 4 in Experiment 2.

GENERAL DISCUSSION

The most important conclusion to be drawn from our results on the effects of operators *not* and *very* on the concepts *small* and *large* is that natural language concepts can be described more completely and manipulated more precisely using the framework of fuzzy set theory. A further, not uninteresting, finding was that concepts and operators can be interpreted in two different ways: what we have called the *linguistic* and *logical* interpretations. This latter finding is independent of any considerations relating to the treatment of concepts as fuzzy sets. It, however, does

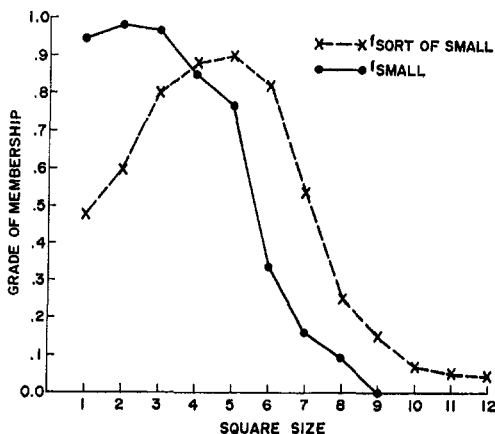


FIGURE 29. Membership function for *sort of small*.

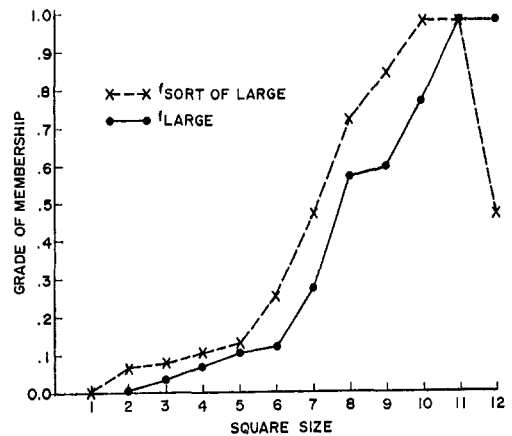


FIGURE 30. Membership function for *sort of large*.

have important consequences for general psycholinguistic theory. Thus, it raises a cautionary note to be heeded by those who would treat certain lexical items as if they were strict logical operators. The results we have reported show that in certain natural language settings entailment does not strictly hold (e.g., *large* is not strictly entailed by *very large*) and that certain combinations of operators will assume an idiomatic sense very different from that predicted by a simple linear combination of the lexical items constituting the phrase (e.g., *not very large*).

As pointed out in the introduction, the implicit claim underlying our work is that natural language concepts are intrinsically vague, a view shared by an increasingly large number of psychologists, linguists, and philosophers. The treatment of natural language concepts as vague has important implications for semantic theory. The major implication is that the meaning of a term could be specified as a *fuzzy set of meaning components*. That is, a specific meaning component need not be necessarily either a member or a nonmember of the set of features that define the meaning of a term. All that is required in this view is that a meaning component have a non-zero degree of membership in the set. This approach, then, avoids a problem that has been the attention of a number of recently published reports where increasing em-

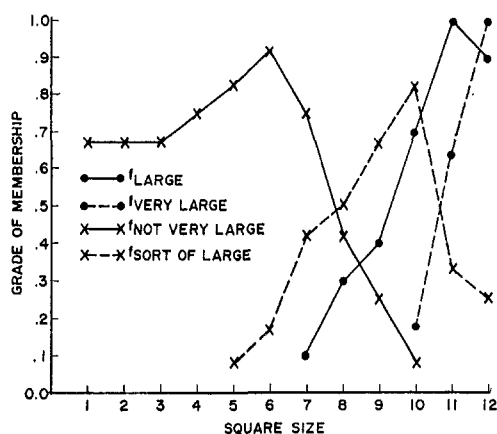


FIGURE 31. Membership functions resulting from the presentation of a single phrase to each subject (Experiment 4).

phasis has been placed on the problem of how to exactly determine what aspects of the meaning of a term are necessary and sufficient to characterize that term (Lehrer, 1970; Slote, 1966). The thrust of a number of empirical reports is that the traditional component approach to meaning (e.g., Katz & Fodor, 1963) incorrectly assumes that a specific component can be said, without any uncertainty, to be necessary for the definition of a term (Caramazza, Grober, & Zurif, in press; Garvey, Caramazza, & Yates, 1975; Lehrer, 1970; Rosch, 1973; Smith, Shoben, & Rips, 1974). More generally, the traditional componential approach is criticized for treating all components as contributing equally to the definition of a term. The alternative proposal emerging from these critical reports is that various components of meaning are differentially important to the definition of a term and, in addition, that no subset of these components can conclusively be said to be necessary and sufficient to define a term: That is, the semantic structure of a term itself is ill-defined, or vague. Though this latter approach may seem to be unnecessarily complicated, it allows for more natural explanations of important classes of semantic phenomena. Thus, for example, Labov (1973) has shown that attempts to give well-defined characterizations in terms of traditional com-

ponential analysis of the semantic structure of a common concept such as *cup* are inadequate. Labov obtained data on the consistency of naming various cuplike objects that support a theory in which various features are differentially weighted as necessary to define the term. The results Labov reported have been replicated in two independent studies with children as subjects (Anderson, 1975; DeVos & Caramazza, Note 4). In the study by DeVos and Caramazza (Note 4), it was shown that perceptual features of objects and functionally determined contexts interact in fairly specific ways to determine the label a child will assign to an object. The implication of this latter study is that from very early in life a child forms concepts that are vague (fuzzy), and contextual cues operate on perceptual features to determine the relative weighting the features are assigned in choosing a label (name) for the object. It is unclear how a traditional feature theory of meaning would handle such findings without recourse to complex ad hoc principles.

The importance of vagueness (fuzziness) as a basic concept in semantic theory can be seen by considering yet another semantic phenomenon: polysemy. Lexicographers have long recognized the fluidity of polysemy: No clear boundary distinguishes polysemy from homonymy. However, traditional feature theory requires that a definite demarcation hold between polysemous and homonymous senses—essentially marked by the presence of a specific feature. In fact, the defining property of polysemous words is that the senses that comprise a word share a common meaning core while at the same time differ sufficiently to be recognized as distinct from each other. In a recent investigation of polysemous words, it was shown, however, that the grade of membership of various senses in a general word concept vary considerably (Caramazza et al., in press). This finding is difficult to interpret within traditional feature theories, since they do not normally allow for features to be only partially applicable. This same finding can be easily interpreted

within a theory that allows "vague" core meanings, where specific senses differ in degrees of membership to that core. Incidentally, it should be pointed out that Katz and Bever (Note 5) have criticized theories that postulate graded membership of instances in categories for confusing performance variability with inherent variability in concepts. They also make the blanket assertion that theories of this sort are based on empiricist principles. While not necessarily disagreeing with their characterization of Rosch's (1973) work, in particular, as having an empiricist flair, we find their wholesale labeling as "empiricist" theories that propose vague concepts to be rather naive. That is, there is no a priori reason to suggest that a rationalist theory necessarily excludes vague concepts.

To conclude, we are proposing that natural language concepts be considered as inherently vague and, specifically, as fuzzy sets of meaning components. We are also proposing that language operators—negative markers, adverbs, and adjectives—be considered as operators on fuzzy sets. Though the research we have reported in this article deals with a very narrow issue, modifying concepts, it does relate nonetheless in a direct way to semantic theory. Specifically, we have tried to give theoretical and empirical justification for the use of a formal system that can handle vagueness. The formal treatment of vagueness is an important and necessary (in our view) step toward a more comprehensive handling of natural language phenomena and the communication of vague information.

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